ECONTUTORIALS · FOR TEACHERS

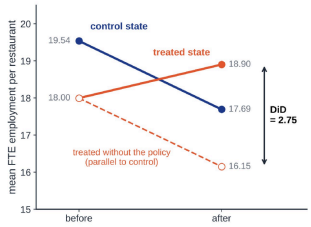
Econometrics Made Simple

TEACHER PACK

20 ready-to-teach chapters with Python, R and Stata labs

SLIDES · LESSON PLANS · TESTS · EXAMS · TEXTBOOK + PRACTICE BOOK INCLUDED

Parallel trends and the counterfactual



- Without the policy the treated state follows the dashed line: 17.998 − 1.846 = 16.152
- Actual 18.902; gap 18.902 − 16.152 = 2.751 = DiD
- Levels may differ; trends must not
- With one pre-period, parallel trends cannot be checked

Lab 2: 2SLS with controls and two instruments

Python

```
from linearmodels.iv import IV2SLS
wages["expersq"] = wages["exper"] ** 2
controls = ("exper + expersq + tenure + female"
            " + married + urban + union")

def tsls(instruments):
    f = ("lwage ~ 1 + " + controls
         + " + [educ ~ " + instruments + "]")
    return IV2SLS.from_formula(f, data=wages).fit(
        cov_type="unadjusted", debiased=True)

iv1 = tsls("dist_college")
iv2 = tsls("dist_college + parent_educ")
```

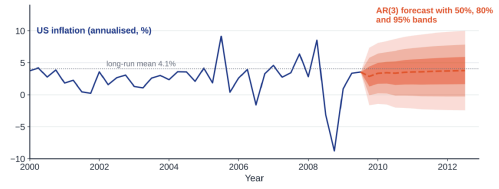
Output

```
Trimmed: educ coefficient
                 OLS   IV dist   IV both
coef          0.1054    0.0612    0.0801
std err       0.0075    0.0448    0.0182
1st F                    29.38    101.73

Hand-made stage 2:
  coef 0.0612  se 0.0482
```

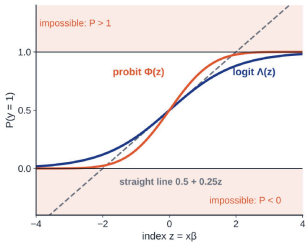
- Brackets [educ ~ z] mark the endogenous variable and instruments
- 2SLS 95% CI with distance: −0.027 to 0.149

A fan chart: AR(3) forecasts of US inflation



- One step: 2.87% (se 2.29); four steps: 3.38% (se 2.74); twelve steps: 3.80% (se 3.16)
- 50%, 80% and 95% bands fan out as the horizon grows; forecasts return to the long-run mean
- The Bank of England has published inflation projections as fan charts since the 1990s

Logit and probit: an S-shaped response



- P(y = 1 | x) = G(xβ), with G a CDF, always between 0 and 1
- Logit: Λ(z) = 1/(1 + e^−z)
- Probit: Φ(z), the standard normal CDF
- Steepest at p = 0.5; flat near 0 and 1

THE ECONTUTORIALS TEAM

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CHAPTER 13

Endogeneity and Instrumental Variables

13

Two ready-to-teach lessons · Econometrics Made Simple

Wooldridge Ch 15–16

Stock & Watson Ch 12

EconTutorials

Learning objectives

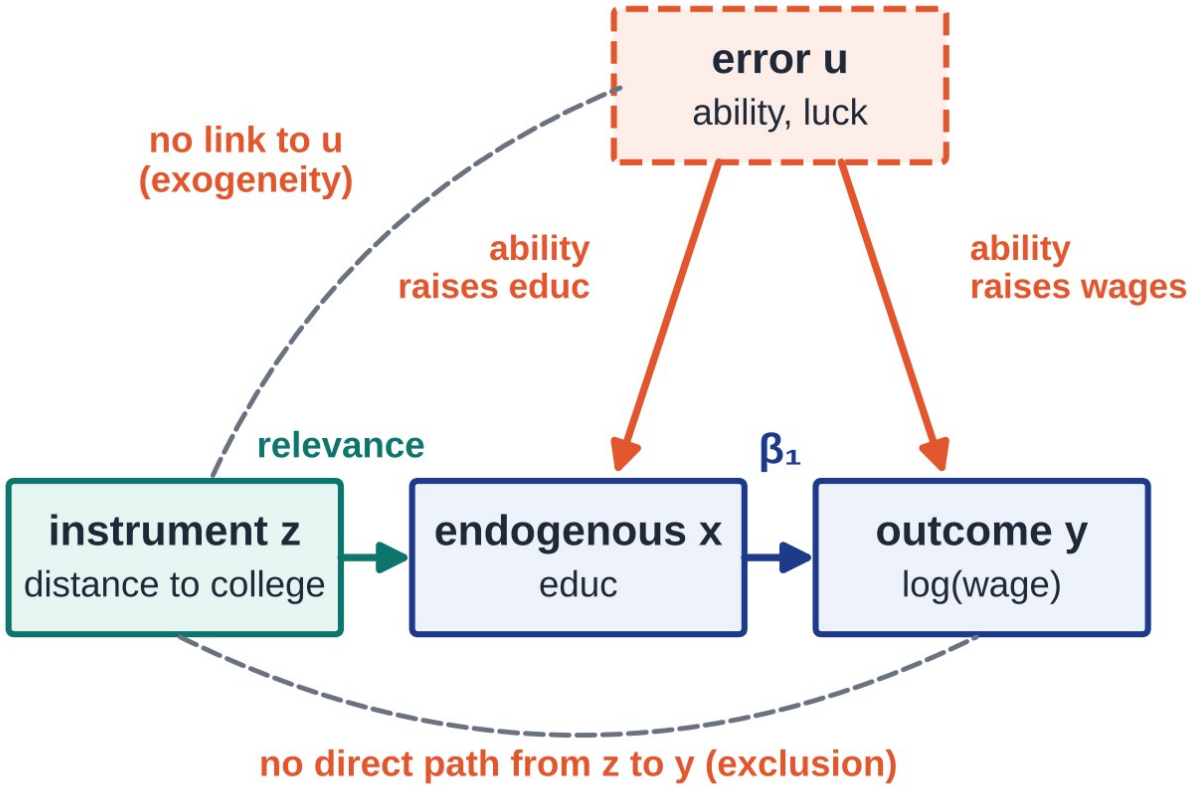
1. Define endogeneity and explain why it makes OLS biased and inconsistent
2. Explain how omitted variables, measurement error and simultaneity create endogeneity
3. State the conditions for a valid instrument and compute IV and Wald estimates
4. Carry out 2SLS with software and explain why hand-made standard errors are wrong
5. Diagnose weak instruments, test endogeneity and overidentification, and interpret IV as a LATE

Syllabus alignment

Wooldridge
Ch 15, 16

Stock & Watson
Ch 12

What makes a valid instrument



- Relevance: z moves x, Cov(z, x) ≠ 0. Testable: regress x on z
- Exogeneity: Cov(z, u) = 0 and no direct path from z to y
- Exogeneity cannot be tested with one instrument: argue it
- Ability (in u) drives schooling and wages, so OLS mixes the red paths

Worked example: the Wald estimate in wages.csv

Split workers at the median distance to college (20.65 km). Mean log wage: near 2.8994, far 2.8685. Mean schooling: near 11.944, far 11.338.

1. Reduced form: 2.8994 − 2.8685 = 0.0309 (about 3.1% higher wages)
2. First stage: 11.944 − 11.338 = 0.606 more years of schooling
3. Wald estimate = 0.0309 ÷ 0.606 = 0.051
4. Continuous instrument gives 0.0709; OLS gives 0.1026

Answer If distance works only through schooling, a year of schooling raises wages by about 5.1%. Splitting at the median throws away information.

Two-stage least squares (2SLS)

1

Stage 1

Regress x on ALL exogenous variables: instruments and controls. Save $\hat{x}$

2

Stage 2

Regress y on $\hat{x}$ and the controls. The slope on $\hat{x}$ is $\hat{\beta}$ 2SLS

!

Standard errors

Hand-made stage 2 uses $\hat{x}$ in the residuals: wrong SEs. Use a 2SLS command

#

Identification

Need at least as many excluded instruments as endogenous regressors

Lab 2: 2SLS with controls and two instruments

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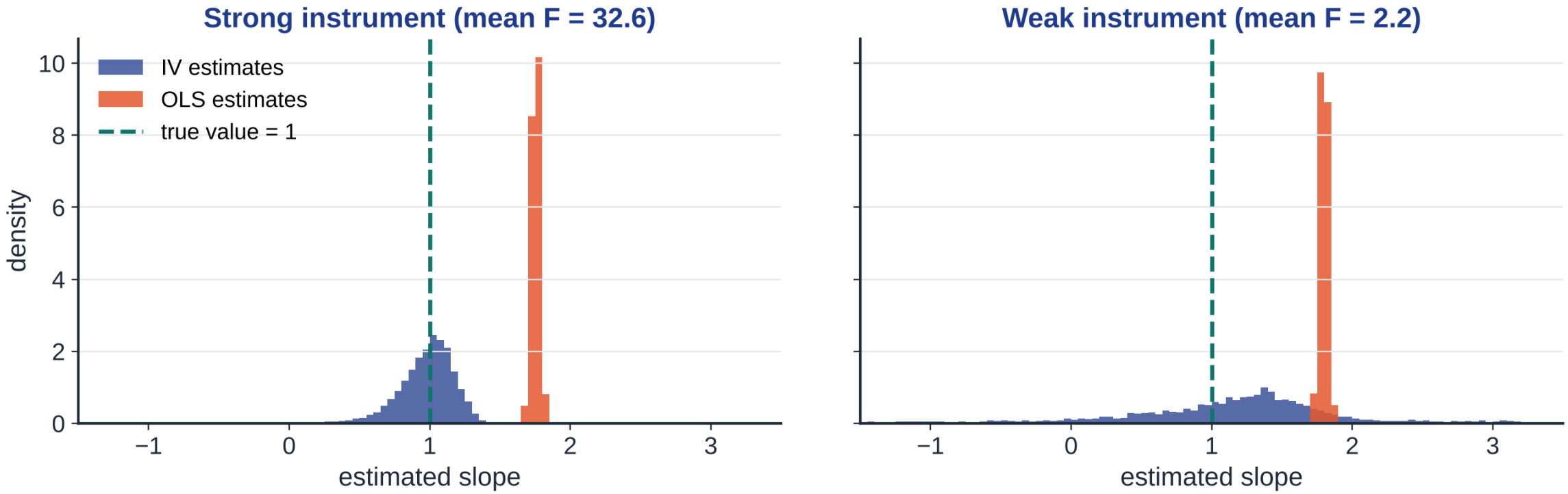
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- Brackets [educ ~ z] mark the endogenous variable and instruments
- 2SLS 95% CI with distance: −0.027 to 0.149

Weak instruments: 5,000 simulated samples



- Strong (mean F = 32.6): IV median 1.005, t-test rejects the truth 5.9% of the time
- Weak (mean F = 2.2): IV median 1.250, pulled towards OLS; rejects 10.4%
- OLS is centred near 1.75, almost twice the true value of 1

Diagnostic tests in the wage equation

Test	H_0	Result in wages.csv
First-stage F	Instruments irrelevant	29.38 (one instrument); 101.73 (two)
Durbin–Wu–Hausman	educ is exogenous	$\hat{v}$ coef 0.0455, t = 1.019, p = 0.308
Sargan (two instruments)	All instruments exogenous	nR^2 = 0.2212, p = 0.638

Failing to reject is not proof: the IV standard error is large, so these tests have low power.

Case study: compulsory schooling and quarter of birth

1. Children born early in the year start school older, so can leave at the legal age with less schooling
2. 1980 US census, 327,509 men born 1930–39: Q1-born men had less schooling and lower wages
3. The authors found IV estimates of the return to education close to OLS

0.102

Wald estimate of the return to a year of schooling (OLS 0.0709)

Discuss Is quarter of birth a valid instrument? Name one channel that could break the exclusion restriction, and say whose return to schooling it measures.

Source: Angrist, J. D. and Krueger, A. B. (1991) "Does Compulsory School Attendance Affect Schooling and Earnings?", Quarterly Journal of Economics 106(4), 979–1014, Table III (also NBER Working Paper 3572).

CHAPTER 13 · LESSON PLAN

Endogeneity and Instrumental Variables

Two 60-minute lessons · Slides: Ch13-Endogeneity-and-Instrumental-Variables-Slides.pptx · Worksheet, test and exit tickets in this pack

University	Introductory Econometrics (year 2): endogeneity, instrumental variables and two-stage least squares. Matches Wooldridge, Introductory Econometrics, Ch. 15 (IV and 2SLS) and Ch. 16 (simultaneous equations), and Stock and Watson, Introduction to Econometrics, Ch. 12.
Master's / dissertation	Any dissertation that makes a causal claim from observational data must explain why OLS may be biased and, where an instrument exists, report the first stage, 2SLS and the diagnostic tests set out in this chapter.
Prior learning	Omitted-variable bias (Chapter 6), t and F tests (Chapters 5 and 7), log functional forms and elasticities (Chapter 8), robust standard errors (Chapter 11)
Resources	Slide deck, student worksheet, laptops with Python, R or Stata (or Google Colab), the companion datasets, exit tickets

Learning objectives

1. Define endogeneity and explain why it makes OLS biased and inconsistent.
2. Explain how omitted variables, measurement error and simultaneity create endogeneity.
3. State the conditions for a valid instrument and compute IV and Wald estimates.
4. Carry out 2SLS with software and explain why hand-made standard errors are wrong.
5. Diagnose weak instruments, test endogeneity and overidentification, and interpret IV as a LATE.

Key vocabulary

Term	Meaning
Endogenous regressor	A regressor correlated with the error term: Cov(x, u) ≠ 0.
Exogenous regressor	A regressor uncorrelated with the error term: Cov(x, u) = 0.
Inconsistency	plim $\hat{\beta}_1 = \beta_1 + \text{Cov}(x, u)/\text{Var}(x)$: the estimate settles on the wrong value however large n is.
Attenuation bias	Classical measurement error in x shrinks the OLS slope towards zero by the reliability ratio $\sigma^2_{x^*}/(\sigma^2_{x^*} + \sigma^2_e)$.
Simultaneity	x and y are determined together, so y also affects x (price and quantity; police and crime).
Instrumental variable	A variable z that is correlated with x (relevance) and uncorrelated with u (exogeneity), affecting y only through x.
Exclusion restriction	The instrument has no direct effect on y: it is excluded from the structural equation.
Wald estimator	With a binary instrument, $\hat{\beta} = (\bar{y}_1 - \bar{y}_0)/(\bar{x}_1 - \bar{x}_0)$: the reduced form divided by

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CHAPTER 13 · STUDENT WORKSHEET

Endogeneity and Instrumental Variables

Show your working. Give numerical answers to 3–4 significant figures and state hypotheses in words. Tasks 3 and 4 need Python, R or Stata and the companion datasets.

Name: _____ Class: _____ Date: _____

Formulas plim $\hat{\beta}_1 = \beta_1 + \text{Cov}(x, u)/\text{Var}(x)$ reliability ratio $= \sigma^2_{x^*}/(\sigma^2_{x^*} + \sigma^2_e)$ $\hat{\beta}$ IV $= \text{Cov}(z, y)/\text{Cov}(z, x)$ Wald $= (\bar{y}_1 - \bar{y}_0)/(\bar{x}_1 - \bar{x}_0)$ $F = t^2$ (one instrument) Sargan $= nR^2 \sim \chi^2(q)$

Task 1: Concepts [6 marks]

(a) Define an endogenous regressor and an exogenous regressor. [2]

(b) For each case, name the source of endogeneity: (i) crime regressed on police numbers across cities; (ii) saving regressed on self-reported income; (iii) wages regressed on training, with motivation unobserved. [3]

(c) State the two conditions for a valid instrument and say which one can be tested. [1]

Task 2: By hand [8 marks]

(a) True income has variance 6 and a classical measurement error has variance 2. The true slope is 0.12. Find the reliability ratio and the probability limit of the OLS slope. [2]

(b) In a sample Cov(z, y) = −0.84 and Cov(z, x) = −12. Compute the IV estimate of β_1. [2]

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CHAPTER 13 · CHAPTER TEST

Endogeneity and Instrumental Variables

Time: 50 minutes · Total: 50 marks · Statistical tables or a calculator may be used

Name: _____ Class: _____ Date: _____

Section A: Multiple choice [10 marks]

Circle one answer for each question.

1. A regressor x is endogenous when

A Var(u | x) depends on x
B x is highly correlated with another regressor
C Cov(x, u) ≠ 0
D the errors are autocorrelated

2. Classical measurement error in an explanatory variable makes the OLS slope

A biased towards zero
B biased away from zero
C unbiased but less precise
D exactly zero

3. True x^* has variance 9, the measurement error has variance 1 and $\beta_1 = 0.5$. OLS converges to

A 0.50
B 0.45
C 0.40
D 0.55

4. With one instrument, which condition can be tested with the data?

A Exogeneity
B The exclusion restriction
C Neither
D Relevance

5. Cov(z, y) = −0.3 and Cov(z, x) = −6. The IV estimate of β_1 is

A 0.05
B −0.05
C 2
D 20

6. A first-stage regression with one instrument gives t = 3. The first-stage F is

A 3
B 1.73
C 9
D 6

7. When the instrument is weak, 2SLS tends to be

A unbiased
B biased towards OLS
C more precise than OLS
D equal to zero

8. In a control-function regression the coefficient on $\hat{v}$ has t = 0.8. The best conclusion is

A x is certainly exogenous
B the instrument is weak
C the instrument is invalid
D no evidence that x is endogenous

9. To estimate a demand curve, the instrument for price must

A shift supply only
B shift demand only
C shift both curves
D be uncorrelated with price

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CHAPTER 13 · CHAPTER TEST MARK SCHEME

Endogeneity and Instrumental Variables

Total 50 marks. Award marks for correct econometrics even where wording or method differs (follow-through marks where shown).

Section A: Multiple choice

Q	Answer	Reason
1	C	Endogeneity means a regressor correlated with the error.
2	A	Attenuation: the slope converges to β_1 times the reliability ratio.
3	B	Reliability ratio 9/10 = 0.9; 0.9 × 0.5 = 0.45.
4	D	Relevance is checked in the first stage; exogeneity involves the unobserved u.
5	A	−0.3/−6 = 0.05.
6	C	With one instrument $F = t^2 = 9$.
7	B	A weak first stage picks up mostly noise, pulling 2SLS towards OLS.
8	D	Failing to reject is not proof of exogeneity; the test may lack power.
9	A	Supply shifters move the equilibrium along the demand curve.
10	B	IV estimates a local average treatment effect for compliers.

Question 11: The return to education by 2SLS [10]

(a) Ability is omitted and raises both schooling and wages B1; so Cov(educ, u) > 0 and OLS is biased upwards B1.

(b) 0.0612 ± 1.96 × 0.0448 M1; = −0.027 to 0.149 A1; contains the OLS value 0.1054 (and zero): the data cannot reject OLS A1.

(c) F = 29.38 > 10: not weak by the Staiger–Stock rule B1; but below about 104.7, so the usual 5% t-test may be distorted; be cautious B1.

(d) H_0: educ is exogenous ($\rho = 0$) B1; t = 1.019 < 1.96, do not reject B1; the IV standard error (0.045) is large relative to the OLS–truth gap, so the test has low power; not rejecting is not proof B1.

Question 12: The demand for coffee [10]

(a) Observed points are equilibria where demand and supply both shift B1; price is correlated with the demand error, so the slope mixes the demand and supply slopes (pulled towards the positive supply slope) B1.

(b) Rainfall shifts supply (growers' costs) and has no obvious effect on consumers' demand at a given price B1; it is strongly relevant (F = 354.2) B1; fuel_cost may also squeeze household budgets and reduce demand directly, breaking exclusion B1.

(c) A 1% price rise reduces quantity demanded by about 0.59% B1; 10% rise: quantity falls about 5.9% M1; revenue rises by about 4.1% because demand is inelastic A1.

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