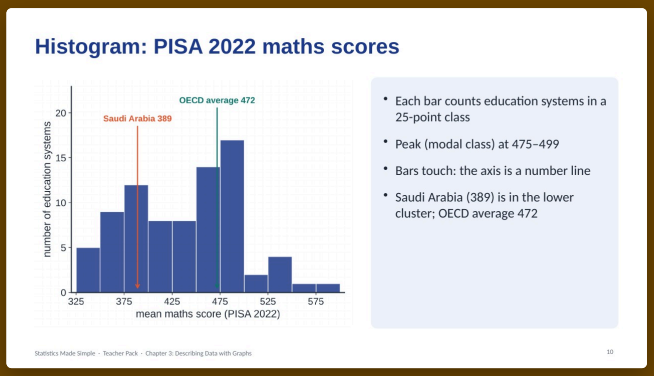
ECONTUTORIALS · FOR TEACHERS

Statistics Made Simple

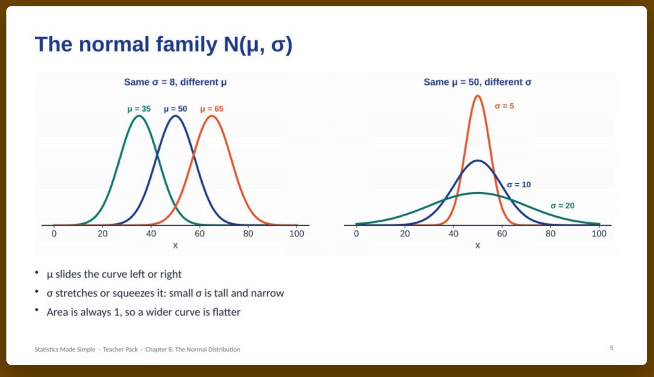
TEACHER PACK

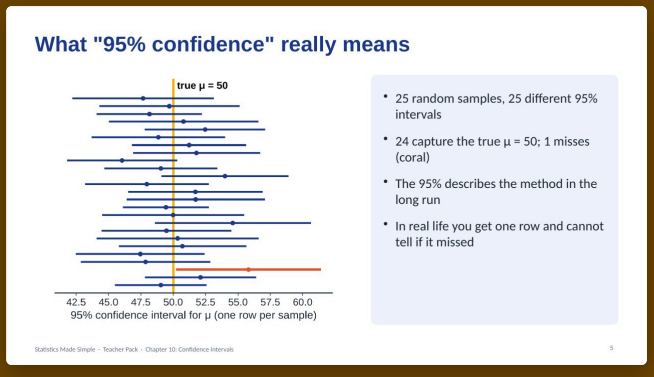
16 ready-to-teach chapters for AP Statistics, Cambridge 9709 and IB

SLIDES · LESSON PLANS · TESTS · EXAMS · TEXTBOOK + PRACTICE BOOK INCLUDED









THE ECONTUTORIALS TEAM

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CHAPTER 8

The Normal Distribution

8

Two ready-to-teach lessons · Statistics Made Simple

AP Statistics Unit 2
Cambridge 9709 5.5, 6.2
IB Maths AA SL 4.9, 4.12

EconTutorials

Learning objectives

1. Explain why probability is area under a density curve
2. Use the empirical rule and standardise with z-scores
3. Find normal probabilities to the left, right and between values
4. Work backwards from a probability to a value, μ or σ
5. Combine normal variables, check normality and approximate the binomial

Syllabus alignment

AP Statistics
Unit 2: normal distribution

Cambridge 9709
Paper 5: 5.5; Paper 6: 6.2

IB Maths AA
SL 4.9 and SL 4.12

Starter: bell-shaped or not?

5 min Sketch what you think the histogram of each variable looks like. Which are roughly bell-shaped?

Heights of adult women

Household incomes

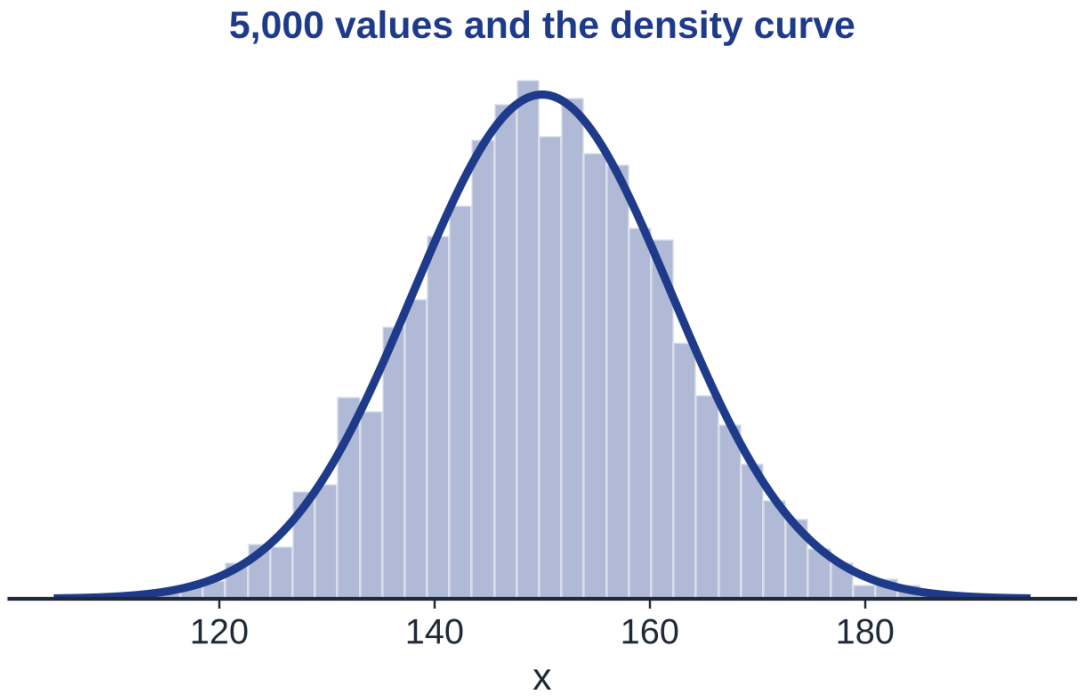
SAT total scores

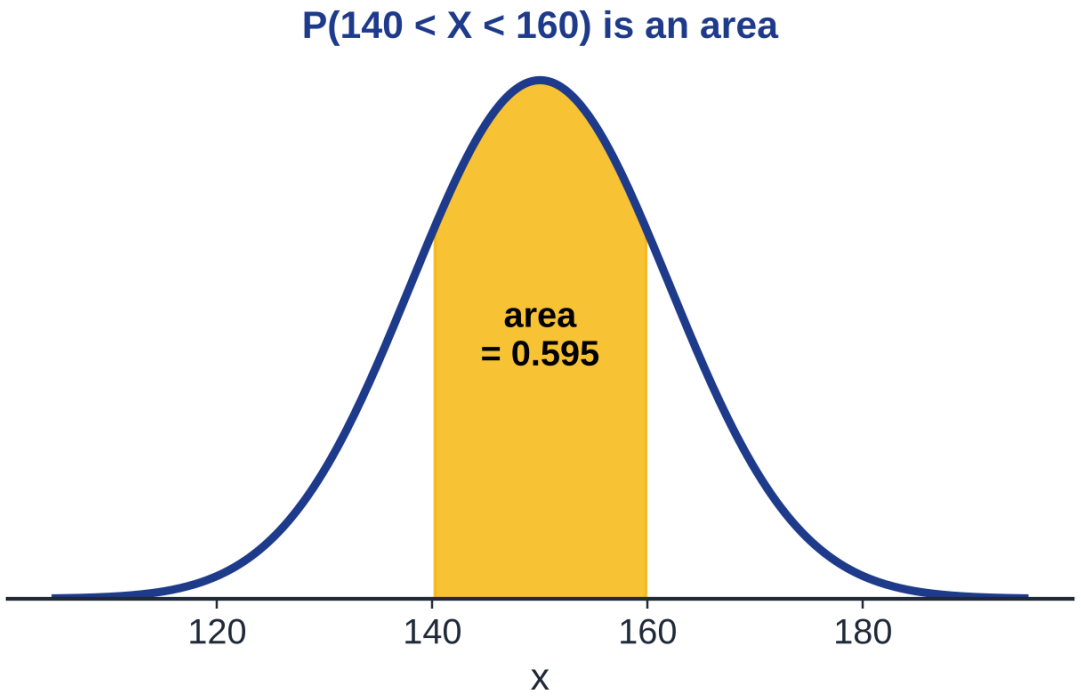
Waiting times at a bus stop

Errors in machine-cut pipe lengths

Children per family

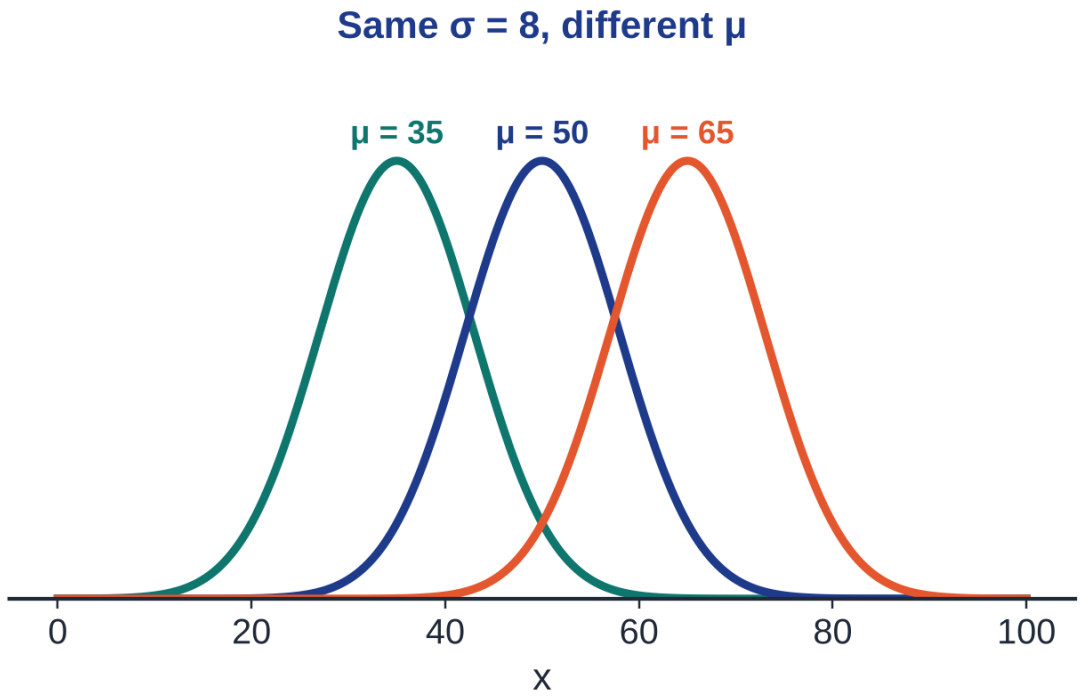
From histogram to density curve

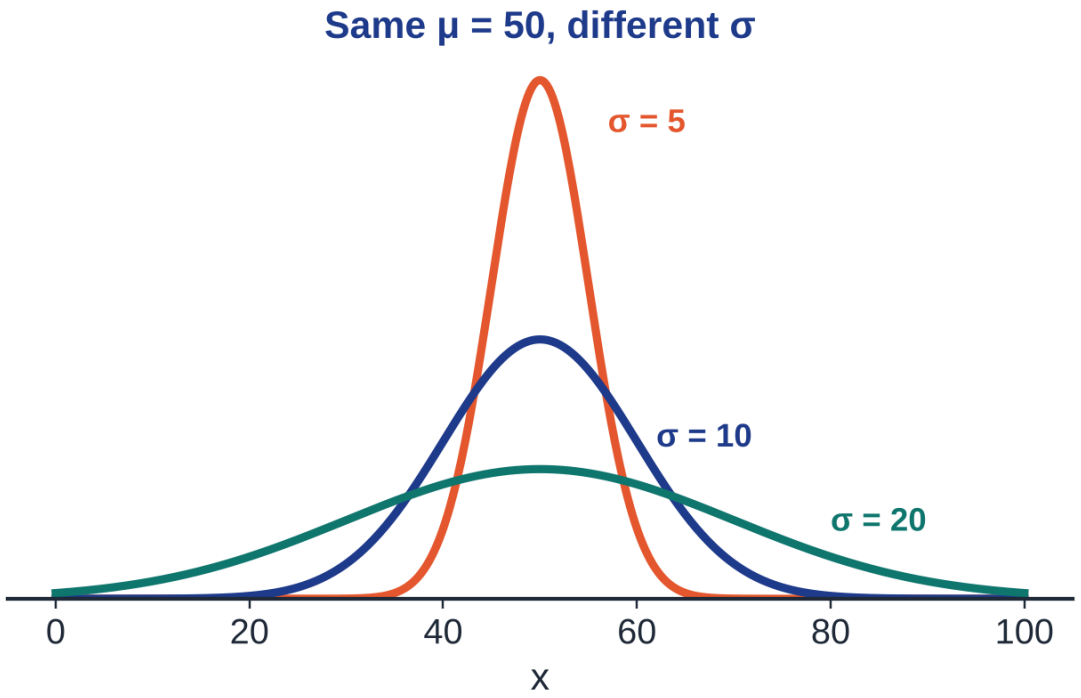




- More data and narrower bars: the outline smooths into a curve
- Total area under the curve = 1
- Probability = area. For continuous X, P(X = a) = 0, so P(X < a) = P(X ≤ a)

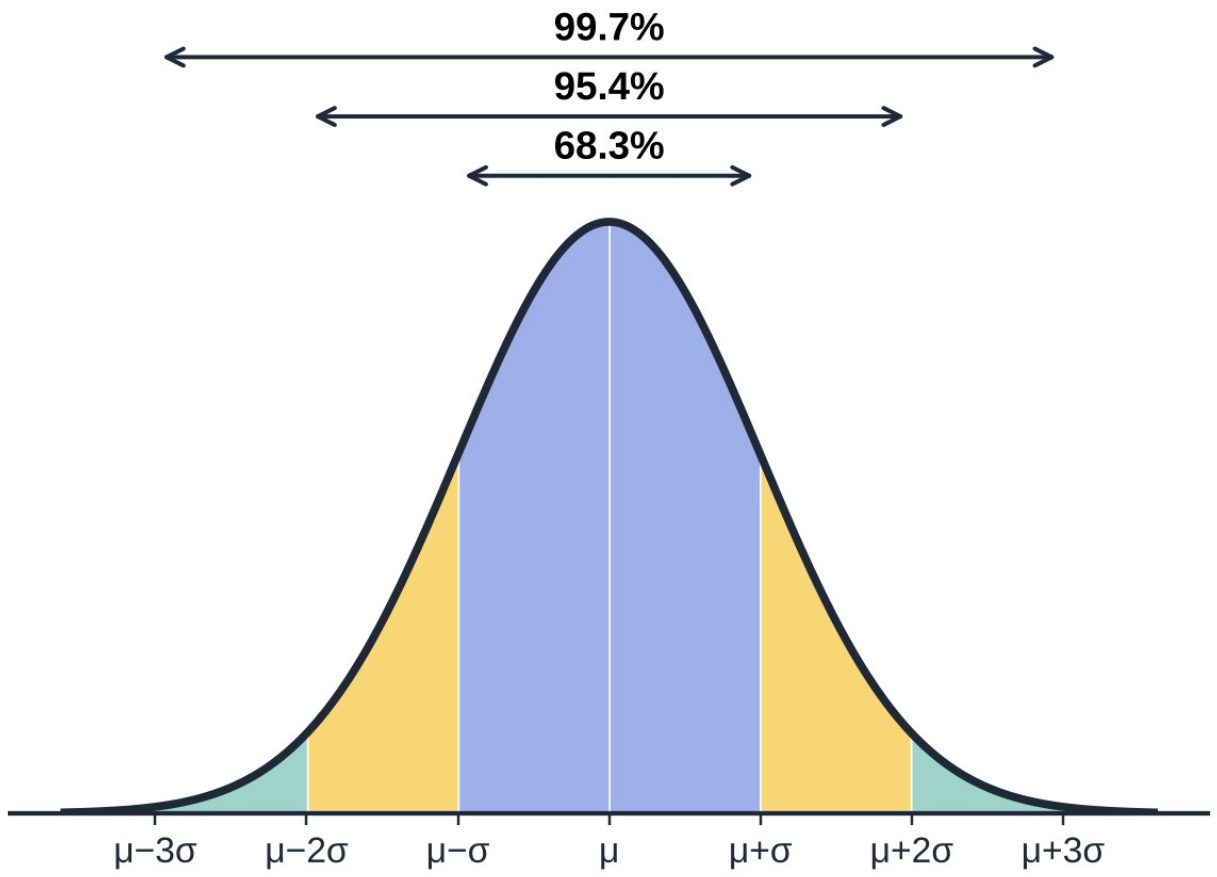
The normal family N(μ, σ)





- μ slides the curve left or right
- σ stretches or squeezes it: small σ is tall and narrow
- Area is always 1, so a wider curve is flatter

The empirical rule: 68–95–99.7



- Within 1σ: 68.3%
- Within 2σ: 95.4%
- Within 3σ: 99.7%
- Quick estimates only at whole numbers of σ

Standardising: z-scores

Key term

$z = (x - \mu) \div \sigma$: the number of standard deviations a value lies from the mean.

$$z = (x - \mu) \div \sigma$$

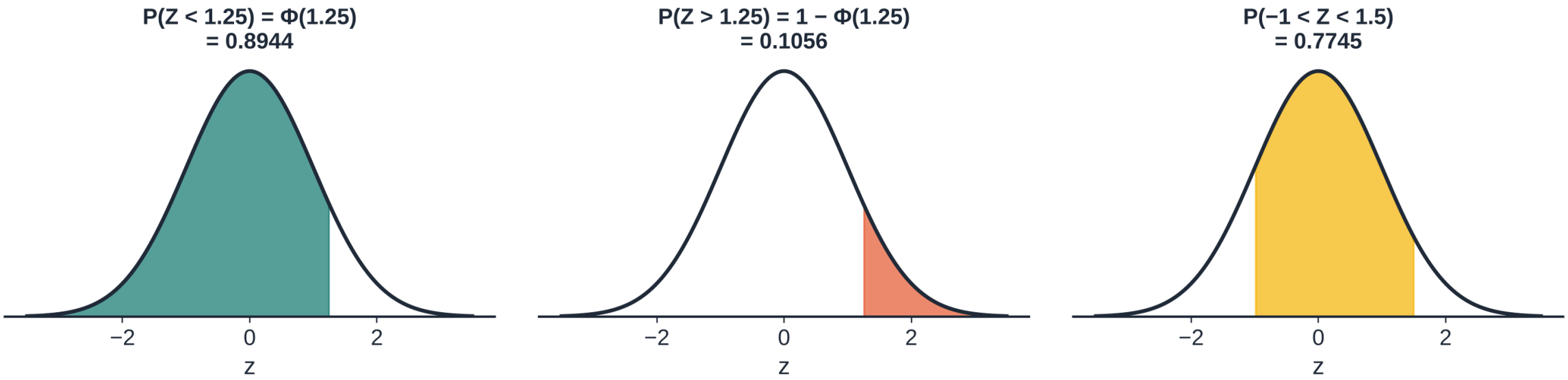
μ

Every normal variable becomes the standard normal N(0, 1) after standardising

σ

Reverse it to find a value: $x = \mu + z\sigma$

Three kinds of area question



- Left: $\Phi(z)$ straight from the table or normalcdf
- Right: $1 - \Phi(z)$
- Between: $\Phi(z_2) - \Phi(z_1)$. Always sketch and shade first

CHAPTER 8 · LESSON PLAN

The Normal Distribution

Two 60-minute lessons · Slides: Ch08-The-Normal-Distribution-Slides.pptx · Worksheet, test and exit tickets in this pack

AP	AP Statistics (2026–27 framework) Unit 2: Probability, Random Variables and Probability Distributions — normal distribution
Cambridge	Cambridge International AS & A Level Mathematics 9709 (2026–2027): Paper 5 topic 5.5 The normal distribution (standardising, finding μ or σ, normal approximation to the binomial with continuity correction); Paper 6 topic 6.2 (sums and differences of independent normal variables)
IB	IB Mathematics: Analysis and Approaches Topic 4: SL 4.9 (the normal distribution and curve, probability and inverse normal calculations) and SL 4.12 (standardization of normal variables, z-values)
Prior learning	z-scores and the empirical rule (Chapter 4); binomial mean and SD (Chapter 7); combining random variables (Chapter 7)
Resources	Slide deck, student worksheet (one per student), mini-whiteboards, calculators, exit tickets

Learning objectives

1. Explain why probability is area under a density curve.
2. Use the empirical rule and standardise with z-scores.
3. Find normal probabilities to the left, right and between values.
4. Work backwards from a probability to a value, μ or σ.
5. Combine normal variables, check normality and approximate the binomial.

Key vocabulary

Term	Meaning
Density curve	A curve on or above the axis with total area 1; the area over an interval is the probability of landing in it.
Normal distribution N(μ, σ)	Symmetric, bell-shaped density curve fixed by its mean μ and standard deviation σ. Cambridge writes $N(\mu, \sigma^2)$.
Empirical rule	About 68%, 95% and 99.7% of a normal distribution lie within 1, 2 and 3 standard deviations of the mean.
z-score	$z = (x - \mu) \div \sigma$: the number of standard deviations a value lies from the mean.
Standard normal	N(0, 1), the distribution of z; one table or calculator function serves every normal problem.
Inverse normal	Finding the value with a given area to its left: $x = \mu + z\sigma$.
Normal quantile plot	Data plotted against the values expected from a normal distribution; a straight pattern suggests normality.

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CHAPTER 8 · STUDENT WORKSHEET

The Normal Distribution

Sketch and shade a normal curve for every question. Give probabilities to 4 decimal places.

Name: Class: _____ Date: _____

_____

_

Formulas $z = (x - \mu) \div \sigma$ $x = \mu + z\sigma$ $X \pm Y \sim N(\mu X \pm \mu Y, \sqrt{(\sigma X^2 + \sigma Y^2)})$ for independent normals $B(n, p) \approx N(np, \sqrt{(np(1 - p))})$

Task 1: The empirical rule [5 marks]

Reaction times of drivers are approximately N(0.75, 0.15) seconds.

(a) Between which two times do about 95% of drivers react?

(b) About what percentage react in more than 0.90 seconds?

(c) Is a reaction time of 1.25 seconds unusual? Explain using a z-score.

Task 2: Normal probabilities [7 marks]

Exam marks are N(68, 12).

(a) Find the probability that a student scores more than 80.

(b) Find the probability that a student scores less than 50.

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(c) Find the probability that a student scores between 60 and 85.

Task 3: Working backwards [6 marks]

(a) Using the exam marks in Task 2, find the mark needed to be in the top 15%.

(b) Apples have masses with SD 15 g, and 20% weigh more than 180 g. Find the mean mass.

(c) Explain why you used $\Phi^{-1}(0.80)$ rather than $\Phi^{-1}(0.20)$ in part (b).

Task 4: Sums and differences [6 marks]

(a) A journey has two independent stages: a bus ride N(25, 6) and a walk N(10, 8), in minutes. Find the probability the journey takes more than 51 minutes.

(b) Cups hold N(240, 5) ml and a machine pours N(225, 6) ml, independently. Find the probability that a cup overflows.

(c) Explain why the SD of the overflow difference is larger than either SD alone.

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CHAPTER 8 · CHAPTER TEST

The Normal Distribution

Time: 50 minutes · Total: 50 marks · Graphing calculator allowed

Name: Class: _____ Date: _____

_____

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Section A: Multiple choice [10 marks]

Circle one answer for each question.

1. For a continuous random variable X, P(X = 5) is

A 0.5 **B** the height of the curve at 5

C 0 **D** impossible to say

2. About what percentage of a normal distribution lies within 2 standard deviations of the mean?

A 95% **B** 68%

C 99.7% **D** 50%

3. X ~ N(80, 5). The z-score of x = 72 is

A 1.6 **B** −8

C −0.625 **D** −1.6

4. Φ(1.2) = 0.8849. P(Z > 1.2) is

A 0.8849 **B** 0.1151

C 0.3849 **D** 0.7698

5. P(−1 < Z < 1) is closest to

A 0.68 **B** 0.34

C 0.50 **D** 0.95

6. X ~ N(100, 15). The value with 90% of the distribution below it is about

A 115.0 **B** 124.7

C 119.2 **D** 110.1

7. In Cambridge notation X ~ N(40, 9). The standard deviation of X is

A 9 **B** 81

C 4.5 **D** 3

8. X ~ N(20, 3) and Y ~ N(15, 4) are independent. The SD of X − Y is

A 1 **B** 5

C 7 **D** 25

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CHAPTER 8 · CHAPTER TEST MARK SCHEME

The Normal Distribution

Total 50 marks. Award marks for correct statistics even where wording or method differs (follow-through marks where shown).

Section A: Multiple choice

Q	Answer	Reason
1	C	A single point has zero area.
2	A	95.4% lies within ±2σ.
3	D	(72 − 80) ÷ 5 = −1.6.
4	B	1 − 0.8849 = 0.1151.
5	A	0.6827.
6	C	100 + 1.2816 × 15 = 119.2.
7	D	$\sigma^2 = 9$, so σ = 3.
8	B	√(9 + 16) = 5.
9	C	High values more extreme than normal: right skew.
10	A	X ≥ 30 starts at 29.5 on the continuous scale.

Question 11: Apples [10]

(a) z = (140 − 152) ÷ 9 = −1.333 (M1); P = 0.0912 (A1). [2]

(b) z = −0.778 and 1.444 (M1); 0.9257 − 0.2184 (M1) = 0.7073 (A1). [3]

(c) x = 152 + 1.645 × 9 (M1) = 166.8 g (A1). [2]

(d) (140 − 152) ÷ σ = −1.645 (M1 B1 for ±1.645) so σ = 12 ÷ 1.645 = 7.30 g (A1). [3]

Question 12: Approximating and combining [10]

(a) np = 30 and n(1 − p) = 90 are both large (B1); mean 30 (B1); SD = √(120 × 0.25 × 0.75) = 4.743 (B1). [3]

(b) P(X > 35) = P(Y > 35.5) (B1 continuity correction); z = (35.5 − 30) ÷ 4.743 = 1.160 (M1); P = 0.1231 (A1). (Exact binomial: 0.1241.) [3]

(c) Mean = 6 × 152 = 912 g (B1); SD = √6 × 9 = 22.05 g (B1). [2]

(d) z = (950 − 912) ÷ 22.05 = 1.724 (M1); P = 0.0424 (A1). [2]

Question 13: Investigation [20]

Marking points (award each where seen; allow follow-through where indicated):

- (a) M1 count ÷ 2,004,965; A2 all six correct (A1 for at least four): 400–590: 0.0262; 600–790: 0.1402; 800–990: 0.3038; 1000–1190: 0.2767; 1200–1390: 0.1783; 1400–1600: 0.0747. [3]

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